APPLICATION OF MCMC – GSA MODEL CALIBRATION METHOD TO URBAN RUNOFF MODELING

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1 Introduction

It is well accepted that the urban stormwater is a significant source of pollution for the receiving systems and its quality is a substantial parameter in urban water management. This pollution results mostly from the erosion caused by the runoff of particulate pollutants accumulated on the watersheds during the dry weather period, and mixed with the sediments eroded in the sewers.

Mathematical and computational modeling seems a necessary decision-making tool for the management of urban drainage systems. Currently, existing models are based on a combination of different lumped conceptual models including simplified mathematical functions that describe the processes of generation and transport of pollutants during rainfall.

However, the difficulty, expensiveness and the uncertainty level of the in situ measurement of urban stormwater pollution generate data that rarely allow a satisfactory calibration and validation of these models.

Furthermore, classical optimization methods that are still used up to date for calibration don't allow neither an estimation of the significance of the obtained optimal parameter set, nor a realistic quantification of models' uncertainty. Thus, the existing urban stormwater quality models are rarely used for practical application.

In order to improve these models and their usefulness, a more robust methodology for calibration and validation of models should be used (or developed). Such methodology should be able to provide both an assessment of the uncertainties in the model's parameter values and an evaluation of the confidence in the model predictions.

In this paper, we test the applicability and the effectiveness of a method for model calibration/validation/sensitivity analysis in urban runoff quality modeling. This method based on the Monte Carlo Markov Chain method MCMC consists of a combination of a general algorithm of statistical inference entitled "Metropolis" and a Global Sensitivity Analysis "GSA" method.

This test will be done using data resulting from a survey conducted on the «Marais» catchment in the center of Paris – France (*Gromaire, 1998*).

2 Model assessment method

In the last decade, great attention has been given to the Bayesian inference for model calibration and uncertainty estimation particularly in the case of complex hydrological models (*Beven & al., 1992, Kuczera & al., 1998, Campbell & al., 1999*). Nevertheless, its application in environmental modeling is very rare.

In this study, a combination of three complementary and independent techniques is used to quantitatively assess the uncertainties associated with the model's parameters as well as the output of the model itself.

1) Model calibration:

The Metropolis algorithm (Tanner, 1996) derived from the "MCMC" family of techniques enables the calculation of the probability distribution of parameters describing parameter uncertainty, and the statistical characteristics of the model output errors.

2) Sensitivity analysis:

A Global Sensitivity Analysis technique derived from the family of "Variance based" method and based on the "Fourier Amplitude Sensitivity Test" (*Saltelli & al., 1999*) will be used. It allows the computation of the contribution of each input factor (parameters, initial conditions, boundary conditions) to the model output's variance, and the quantitative estimation of the interaction among model's parameters.

3) Model output uncertainty assessment:

On the basis of the parameters' distributions, the Monte Carlo method determines the conceptual model confidence intervals reflecting its prediction capacity.

3 Site Description

6 street sub-catchments for which 83 suspended solids pollutographs are available are used for this application. Table 1 presents the characteristics of the street sub-catchments.

Table 1: Characteristics of the 6 street sub-catchments

Street name	*Duval*	*Rosiers*	*M.B.M.*	*Roi de Sicile*	*St. Antoine*	*Turenne*
Surface (m^2)	160	186	195	284	1017	1700
Commercial /circulation level	low/low	high/med	no/low	low/low	high/high	Low/med
Nb of pollutographs	11	20	19	10	12	11

* Note that the 6 sub-catchments have approximately same impermeability and same slope

4 Model description

The application consists of testing the existing accumulation and erosion models for 2 kinds of initial conditions (presence or absence of a residual mass from last storm event). The most commonly used model simulates both:

- The accumulation of pollutants during dry weather period which is assumed to follow a linear (eq 1) or an asymptotic behavior(eq 2) and which depends on two parameters : an accumulation rate Daccu and either a dispersion rate Dero or a maximum accumulated mass Mmax = Daccu / Dero.

Linear: $$\frac{dMa(t)}{dt} = Daccu \cdot Simp \text{ and } Ma(t) \leq M_{\max} \quad (1)$$

Asymptotic: $$\frac{dMa(t)}{dt} = Daccu \cdot Simp - Dero \cdot Ma(t) \quad (2)$$

Where Ma(t) (kg) is the available pollutants' mass at time t and S_{imp} (ha) is the impervious area.

- And the erosion of pollutants by runoff during wet period (eq 3) which is supposed to depend on the available pollutants' mass, the runoff and two calibration parameters Wero and w.

$$C(t) = \frac{1}{q(t)} \cdot \frac{dMa(t)}{dt} \text{ and } \frac{dMa(t)}{dt} = -Wero \cdot X(t)^w \cdot Ma(t) \quad (3)$$

Where C(t) (mg/l) is the pollutant concentration produced by erosion, X(t) is either the discharge q(t) (m3/s) at the outlet of the watershed at time t or the rainfall intensity i(t) (mm/hr).

5 Results

The preliminary result shows that the Metropolis algorithm converged successfully to the same posterior probability distribution of the parameters whichever the initial parameter set used. Figure 1 presents the posterior probability distribution obtained for the parameters Daccu, Dero, Wero, W with the Metropolis algorithm for the Duval catchment. The analysis of the posterior distributions of the parameters indicates, furthermore, a strong correlation between the values of the accumulation model parameters (correlation = 0.66) and between the values of the erosion model parameters (correlation = 0.86).

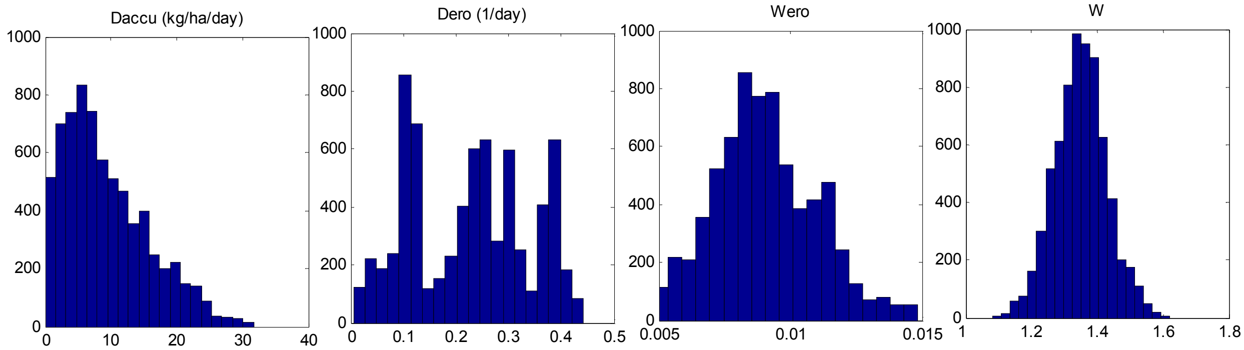

Figure 1 Probability distribution of the accumulation model parameters Daccu & Dero and the erosion model parameters Wero & W

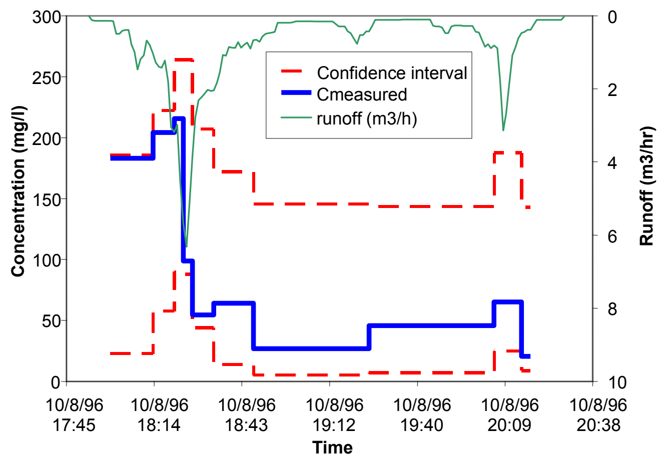

Figure 2 The 5/95 % confidence interval and measured concentration at the outlet of Duval catchment for a rain event

Figure 2 presents the confidence interval obtained by application of Monte Carlo to the model with the probability distribution of parameters. It shows large uncertainties in the model outputs. To explain this, a sensitivity analysis will be performed in this study to determine the relationships between both the model input uncertainties and the model output uncertainties. However this method delivers many information which would have been unreachable with classical calibration methods and which are very useful for modeling attempts.

6 References

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